

Sawtooth Period Scaling

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We discuss the role of neoclassical resistivity and local magnetic shear in the prediction of the Sawtooth period in tokamaks. When collisional detrapping of electrons is considered the value of the safety factor on axis, $q(t, 0)$, evolves on a new time scale, $\tau_* = \tau_\eta \nu_*/(8\sqrt{\epsilon})$, where $\tau_\eta = 4\pi a^2/[c^2 \eta(0)]$ is the resistive diffusion time, $\nu_* = \nu_e/(\epsilon^{3/2} \omega_{te})$ the electron collision frequency normalised to the transit frequency and $\epsilon = a/R_0$ the tokamak inverse aspect ratio. Such evolution is characterised by the formation of a structure of size $\delta_* \sim \nu_*^{2/3} a$ around the magnetic axis, which can drive rapid evolution of the magnetic shear and decrease of $q(t, 0)$. In this paper we investigate two possible trigger mechanisms for a Sawtooth collapse corresponding to crossing of the linear threshold for the $m = 1$, $n = 1$ instability and non-linear triggering of this mode by a core resonant mode near the magnetic axis. The Sawtooth period in each case is determined by the time for the resistive evolution of the q profile to reach the relevant stability threshold. In both cases we focus on the scaling of the Sawtooth period as ν_* is varied.

I. INTRODUCTION

When the safety factor, q , falls below unity, tokamaks experience a periodic oscillation in the plasma core in which core parameters exhibit a sawtooth-like waveform, with a relatively slow ramp-up of, for example, the electron temperature, followed by a rapid collapse. Understanding the characteristics of these Sawtooth oscillations is important for predicting the performance of ITER since they can degrade the core confinement, expel the alpha particles that heat the burning plasma and couple to other instabilities that can severely limit operation. The length of the sawtooth period, which is terminated by the sawtooth collapse, plays a major part in determining the impact of the sawtooth on the tokamak performance. Following a sawtooth collapse, the radial profiles of the various plasma parameters evolve on transport timescales until a rapidly growing instability is triggered, producing magnetic reconnection on a fast timescale. It is this evolution, on the relevant transport time scale, that determines the sawtooth period. However the actual triggering event remains to be identified. In this paper we explore two potential avenues.

The first, considered in Section 2, is in the spirit of the model proposed by Porcelli, Boucher and Rosenbluth [1] which, in particular, proposes a condition on the magnetic shear at the $q = 1$ surface for triggering the sawtooth collapse. This condition is loosely connected with diamagnetic stabilisation of the internal kink mode. Here, we establish more rigorously the stable window in an operating diagram for the $m = 1$, $n = 1$ drift-tearing mode and the resistive internal kink mode, defined in terms of the plasma beta $\beta = 8\pi p/B^2$ (the ratio of kinetic and magnetic plasma pressure), and the instability drive, represented by a quantity Δ' which is inversely related to the potential energy for the internal kink mode δW . This diagram results from an earlier study [2] of the stability of these two modes based on a plasma model with semi-collisional electrons and ions whose Larmor orbit exceeds the semi-collisional layer width where reconnection occurs. The evolution of the location of the core plasma state in this diagram, as the profiles of q and plasma pressure change during the sawtooth ramp, can be monitored and the triggering of the sawtooth crash identified as the point at which the linear stability threshold is crossed. This theory also predicts that the crash occurs when the magnetic shear, $\hat{s} = rq'/q$, at $q = 1$ reaches a particular value, but is much more precisely defined than in the model of Porcelli et al. [1]. In particular, the electron temperature gradient, through the parameter $\eta_e = d\log(T_e)/d\log(N_e)$, is a significant parameter. Using a neoclassical conductivity the resistive evolution of the shear at $q = 1$ is monitored following a Kadomtsev reconnection of the q profile as a result of the sawtooth crash. The time to reach this critical shear then determines the sawtooth period; we deduce a scaling law for this.

The second avenue, discussed in Section 3, builds on ideas of Park and Monticello [3] who realised that the resistive evolution of the q profile in neoclassical theory leads to a rapid, cusp-like drop of q on axis due to the effect of trapped electrons, with q rapidly falling to $q \sim 0.8$ in a typical MHD simulation. A more accurate treatment includes the collisional correction at small $\nu_{*e} = \nu_e/\epsilon^{3/2} \omega_{te}$ (here ν_e is the electron collision frequency, $\omega_{te} = v_{the}/Rq$ the electron transit frequency and $\epsilon = a/R$ the inverse aspect ratio). This removes the trapped particle cusp behaviour very close to the axis so that dq/dr becomes zero there, although the value of q is still rapidly driven well below unity. In this case an alternative "trigger" for the Sawtooth collapse is taken to be $q(0) < q_{crit}$ with $q_{crit} \sim 0.75$, or a similar value. The justification for such a simple criterion lies both in experimental data which tend to show, in many devices, such as TEXTOR, JET, and TFTR, for example, minimum values of $q_0 \sim 0.7$ just prior to a sawtooth collapse; and partly from the theoretical expectation that as the value of q_0 falls significantly below unity, new resonant MHD modes

become possible in this region of unfavourable average curvature: i.e. that the plasma core is increasingly at risk of undergoing a Taylor relaxation of the type suffered in Reversed Field Pinch devices [4]. We have investigated the consequences of this for the sawtooth period, based on the assumption that a core instability might occur when q_0 falls to some critical value below 0.75, say, when the $m = 3, n = 4$ ideal or tearing mode might be destabilised, for example. Such an unstable mode can couple toroidally to a mode resonant at $q = 1$, thus driving the $m = 1, n = 1$ instability, which is invariably observed during the sawtooth collapse. Again the time for q to evolve to such an axial value provides a prescription for the sawtooth period for which we deduce again a scaling law.

The scaling laws for the sawtooth periods resulting from these two models and their implications for ITER are discussed in the Conclusions in Section 4

II. LINEAR STABILITY AND THE SAWTOOTH TRIGGER

In Ref. [2] we developed a unified theory of the drift-tearing mode and internal kink mode relevant to the $m = 1, n = 1$ mode resonant at $q = 1$ in large hot tokamaks such as ITER. Specifically, we adopted a plasma model with semi-collisional electrons for which:

$$k_{\parallel}^2 v_{the}^2 \sim \omega \nu_e \quad (1)$$

with $k_{\parallel} \equiv k_y x / L_s$, k_y the component of the perpendicular wavenumber lying within the magnetic surface, x the distance from the resonant surface and $L_s = Rq/s$, the shear length. The resulting width of the electron current channel δ is thus given by:

$$\delta = \frac{(\omega \nu_e)^{1/2} L_s}{k_y v_{the}}. \quad (2)$$

We consider the ion Larmor orbit to be large, $\rho_i \gg \delta$. The theory is characterised by two other key parameters: $\hat{\beta} = 0.5 \beta_e L_s^2 / L_n^2$ and $\Delta' = \hat{s}_1^2 / \delta W$, where $L_n^{-1} = -N_e^{-1} \partial_r N_{0e}$ is the inverse of the equilibrium density gradient length, and Δ' is the instability drive [5]. For the $m = 1, n = 1$ modes, this is related to the potential energy of the internal kink mode, δW . The key results are that, at low values of $\hat{\beta}$, the drift-tearing mode, with frequency $\omega = \omega_{*e}(1 + 0.73\eta_e)$, is stabilised by finite ion orbit and diamagnetic effects, provided that [see Eq. (42) of [2]]:

$$\Delta' \rho_i < \Delta_1, \quad (3)$$

where

$$\Delta_1 = \sqrt{\pi} \hat{\beta} \frac{(\hat{\omega} - 1)^2 (\hat{\omega} + 1) (\hat{\omega} + 1 - \eta_i/2)}{4 \hat{\omega}^2} \log(\Lambda) - \pi \hat{\beta} (\hat{\omega} - 1) \bar{I}, \quad (4)$$

with

$$\Lambda = e^{-\frac{\pi}{4}} \frac{\rho_i}{\delta \hat{\omega}^{1/2}}, \quad (5)$$

$$\hat{\omega} = 1 + 0.73\eta_e, \quad (6)$$

$\hat{\omega} = \omega / \omega_{*e}$, $\omega_{*e} = 1 / 2 k_y v_{the} \rho_e / L_n$, \bar{I} is an integral defined in [2] with approximate value $\sim -\eta_e^{1/2}$, and we have taken $T_e = T_i$ for simplicity. This result is valid at small $\Delta' \rho_i \sim \hat{\beta}$. At higher values of $\Delta' \rho_i$, i.e. as $\hat{\beta} \Delta' \rho_i \sim 1$ the unstable drift-tearing mode couples to a stable Kinetic Alfvén Wave (KAW) until, at a critical value of the parameter $\hat{\beta} \Delta' \rho_i$, there is an exchange of stability, with the drift-tearing mode continuing at the same frequency but now stable, whereas the (previously stable) KAW becomes unstable. With continuing increase of $\hat{\beta} \Delta' \rho_i$ the KAW frequency drops towards $\omega / \omega_{*e} = 1$ and its growth rate also decreases, the mode eventually becoming stable when

$$\Delta' \rho_i > \Delta_2, \quad (7)$$

with

$$\Delta_2 = \frac{\rho_i}{\delta_0} \frac{2.7 \eta_e \hat{\beta}}{\sqrt{1 - 0.1 \eta_e}} \quad (8)$$

where, $\delta_0 = \sqrt{\omega_{*e} \nu_e} r_1 L_s / v_{the}$, and r_1 is the position at which $q = 1$. These results pertain to low $\hat{\beta}$, however, it has also been proved that the KAW is stable when $\hat{\beta}$ exceeds a critical value depending on η_e , due to the effect of shielding

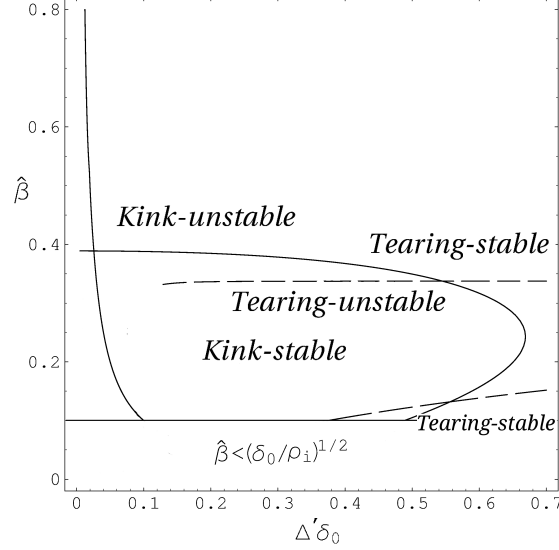


Figure 1: Stability boundary for the drift-tearing mode and kink mode as derived in Ref. [2]. Here $\delta_0/\rho_i = 0.01$.

of the resonant surface by plasma gradients. For the particular case, $\eta_e = 2.53$, this threshold is $\hat{\beta} = 0.34$. Such a result is consistent with that of Drake et al.[6] who found stability in the limit $\hat{\beta} \sim (\hat{\omega} - 1)^{-1} \gg 1$.

In Ref. [2], the stability limits for the dissipative internal kink mode were also determined. In particular, for $\delta W < 0$, one stability limit is given by,

$$\Delta' \rho_i < \Delta_3, \quad (9)$$

where

$$\Delta'_3 \delta_0 = \hat{\beta} \frac{14}{\sqrt{4.08 - 1.71\eta_e}} \frac{1}{\log \left[\hat{\beta}^2 \frac{\rho_i}{\delta_0} \frac{\pi}{2^{3/2}} \sqrt{\frac{4.26}{4.08 - 1.71\eta_e}} \right] + \pi - \frac{1}{2}} \quad (10)$$

[see Eq. (96) of Ref. [2], here we are using $\eta_i = 0$, hence $I_1 = 1/2$], while a general stability boundary for arbitrary $\hat{\beta}$, which ensured stability when $\hat{\beta} > \hat{\beta}_2 \approx \sqrt{\delta_0/\rho_i}$, was also determined.

The combined effects of these stability boundaries is encapsulated in Fig. 1. In particular, when $\hat{\beta} \ll 1$, we observe the existence of two stable ranges for the stability index $\Delta' \rho_i$; namely

$$\Delta' \rho_i < \Delta_1, \quad \Delta'_2 < \Delta' < \Delta'_3. \quad (11)$$

Thus, as Fig. 1 clearly shows, we can identify a new window in $\hat{\beta}$ for the instability for the drift-tearing mode that was missed by previous calculations that exploited arbitrarily large $\hat{\beta}$.

It is crucial to understand the relative magnitude of Δ'_2 and Δ'_3 . In particular, for $\Delta'_2/\Delta'_3 < 1$, the low- $\hat{\beta}$ window of stability is accessible to the system. If we define $f(\eta_e) = \Delta'_2/\Delta'_3 - 1$, we can solve for $f(\eta_e) = 0$, finding that the window of stability is present when $f(\eta_e) < 0$.

In Fig. 2 we show the plot of $f(\eta_e)$; we see that there is a critical electron temperature gradient for which

$$\Delta'_2/\Delta'_3 \leq 1, \quad \text{when } \eta_e \leq \eta_e^{(0)}. \quad (12)$$

Some values of the new critical electron gradient $\eta_e^{(0)}$ are given in Table I.

From Eq. (96) of Ref. [2], it is easy to show that

$$\hat{s}_{crit}^3 \approx |\delta W| \frac{R_0}{a} \frac{r_1}{a} \sqrt{\frac{\Omega_e}{0.5\nu_e}} 2\pi^2 \frac{\beta_e}{\sqrt{4.08 - 1.71\eta_e}}, \quad (13)$$

where \hat{s}_{crit} is the critical shear at $q = 1$, and Ω_e the electron cyclotron frequency. This result can be compared with the result of Ref. [1], where a condition on the shear at the resonant surface is calculated from the inequality

$$\gamma_K > \omega_{*i}, \quad (14)$$

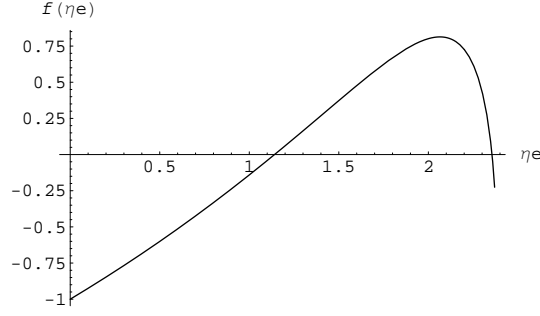


Figure 2: Function $f = f(\eta_e)$ for $\hat{\beta}^2 \rho_i / \delta_0 = 10$.

$\eta_e^{(0)}$	1.161	1.141	1.067	0.987
$\hat{\beta}^2 \rho_i / \delta_0$	9	10	15	25

Table I: Critical electron temperature gradient $\eta_e^{(0)}$ as a function of $\hat{\beta}^2 \rho_i / \delta_0$, given by the solution of the equation $f(\eta_e) = 0$.

where γ_K is the linear growth rate of the mode and ω_{*i} is the ion diamagnetic frequency. The main difference between Eq. (13) and previous results is the explicit functional relation between critical shear and the energy δW , and the precise electron temperature gradient dependence [20].

Once all the boundaries of marginal stability of these modes are determined, it remains to understand how they can be crossed. Now, noting the relationship [7]

$$\Delta' r_1 = \frac{\hat{s}_1^2}{\delta W}, \quad (15)$$

with $\delta W = (r_1^2 / R_0^2) \delta \tilde{W}_T$, where $\delta \tilde{W}_T$ is the energy calculated by Bussac et al. [8], \hat{s}_1 is the shear at the $q = 1$ surface. The shear dependence of the key parameters $\hat{\beta}$ and δ , reveals great sensitivity of the various thresholds to an evolving $q(r)$ profile. In particular, assuming a parabolic pressure profile $p = p_0(1 - r^2/a^2)$ and $T_i = T_e$, (see Eq. (109) of Ref. [2]), for a given inverse aspect ratio ϵ , we have

$$\hat{\beta} = \frac{\beta_0}{\epsilon [aq'(r_1)]^2} \approx 0.5/[aq'(r_1)]^2, \quad (16)$$

in JET or ITER. While it is desirable to run simulations of the Sawtooth cycle that couple stability criteria and transport evolution of all plasma profiles, in this work we content ourselves with the analysis of the post-crash, neoclassical q evolution.

III. EVOLUTION OF THE $q(t, r)$ PROFILE DURING THE SAWTOOTH RAMP

In this Section, we study the importance of the trapped particle correction to Spitzer resistivity in determining the duration of the Sawtooth ramp. During this period, which follows a collapse event, thermal equilibrium can be assumed to be rapidly re-established, but the current profile, and the q profile, evolve resistively towards a remote (and ideal MHD unstable) steady-state in which the toroidal current is $J_\phi = E_0/\eta(r)$, with E_0 constant, and $\eta(r)$ the resistivity. For the moment we ignore the effect of the Bootstrap current within Ohm's law. Neoclassical resistivity is given approximately by [9, 10]

$$\eta(r) = \eta_{Sp}(r)/(1 - \sqrt{r/R_0})^2, \quad (17)$$

where η_{Sp} is the Spitzer resistivity. With the electron temperature profile given by $T_e(r) = T_0(1 - r^2/a^2)^{4/3}$, the Spitzer resistivity has the form

$$\eta_{Sp}(r) = \frac{\eta_0}{(1 - r^2/a^2)^2}. \quad (18)$$

We construct the relevant diffusion equation for the q profile in the cylindrical tokamak limit retaining one toroidal effect, namely the neoclassical correction to resistivity. Thus,

$$\frac{\partial B_\theta}{\partial t} = -c(\nabla \times \mathbf{E})_\theta = c \frac{\partial}{\partial r}(\eta J_z) = \frac{\partial}{\partial r} \left[\frac{\eta c^2}{4\pi r} \frac{\partial}{\partial r}(r B_\theta) \right], \quad (19)$$

and using the definition of the safety factor,

$$q(r) = \frac{r}{R_0} \frac{B_z}{B_\theta}, \quad (20)$$

this becomes

$$\frac{\partial}{\partial \tau} \left(\frac{1}{q} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left[\hat{\eta} \frac{\partial}{\partial r} \left(\frac{r^2}{q} \right) \right], \quad (21)$$

where we have introduced the dimensionless variables defined by $\tau = t/\tau_\eta$, and $r = r/a$, with $\tau_\eta = 4\pi a^2/(\eta c^2)$. The model for neoclassical resistivity is

$$\hat{\eta}(r) = \frac{1}{(1-r^2)(1-\sqrt{\epsilon}r^{1/2})^2}, \quad (22)$$

where $\epsilon = a/R_0$. Clearly, the fractional power in the trapped electron correction to Spitzer resistivity generates (unphysical) singular behaviour in Eq. (21), for $r \rightarrow 0$, i.e. in the vicinity of the magnetic axis. This is removed by including the transition from a neoclassical resistivity to Spitzer when

$$\nu_e > \frac{v_{the}}{R_0 q} \left(\frac{r}{R_0} \right)^{3/2}. \quad (23)$$

Incorporating this correction, the expression for the resistivity becomes

$$\hat{\eta}(r) = \frac{1}{(1-r^2)(1-\frac{\sqrt{\epsilon}r^2}{r^{3/2}+\nu_*})^2}, \quad (24)$$

where $\nu_* = \nu_e/(\epsilon^{3/2}\omega_{te})$. In large tokamaks such as JET or ITER, the dimensionless parameter ν_* is extremely small, so that resistive evolution in the vicinity of the magnetic axis, though not singular there, is likely to be rapid: this will become evident from our numerical solution of Eq. (21). Furthermore, although the scaling of the resistive diffusion time, $\tau_\eta \propto a^2 T_e^{3/2}$ points to a much slower evolution of $q(t, r)$ in ITER than in JET (possibly by a factor of ~ 60), the scaling of the small parameter, $\nu_* \propto N_e a/T_e^2$ reduces this factor when considering core evolution times. For example, by expanding Eq. (21) locally around $x = 0$, and employing Eq. (24), one obtains the solution

$$q_0(t) = q_0(0) \exp(-t/\tau_*), \quad (25)$$

with

$$\tau_* = \tau_\eta \frac{\nu_*}{8\sqrt{\epsilon}} \propto \frac{R_0^3 N_e}{T_e^{1/2}}. \quad (26)$$

Hence, at early times, the safety factor undergoes an exponential decay on the timescale τ_* . Note that the presence of the short timescale τ_* is the consequence of the formation in the q profile of a boundary layer of width $\delta_* \sim \nu_*^{2/3} a$, which we assume to be destroyed by the crash itself, and thus not being present at $t = 0$, is developing only afterwards. We might suppose that a scenario with incomplete reconnection [11], cannot be affected by this time scale, lacking a mechanism to destroy the structure of width $\delta_* \sim \nu_*^{2/3} a$.

A. Evolution from a fully-reconnected Kadomtsev-like state

As an initial $q(r)$ state we first choose the fully reconnected Kadomtsev-like form. This is given by the analytical formula [21]

$$\frac{2}{q_{pc}} = \frac{1 - \tanh[(r - r_{mix})/\zeta]}{q_K} + \frac{1 + \tanh[(r - r_{mix})/\zeta]}{q_{in}}, \quad (27)$$

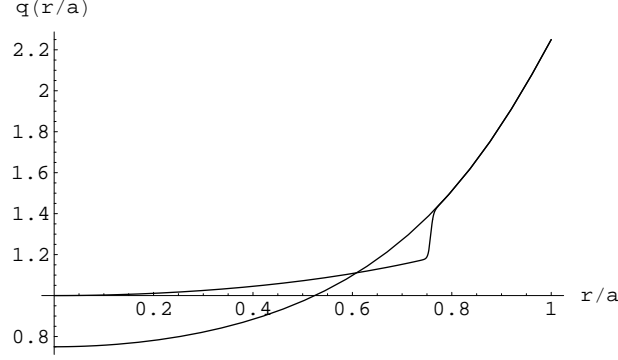


Figure 3: Kadomtsev-like pre-crash $q_{in} = q_0/[1 - r^2/a^2 + (1/3)r^4/a^4]$ and post-crash q_{pc} profile as given by Eq. (27).

where $q_K = 1/(1 - 0.27r^2)$ is the Kadomtsev full-reconnected state which has been calculated numerically for initial $q_0 = 0.75$, $q_{in} = q_0/[1 - r^2 + (1/3)r^4]$ has been chosen for the "pre-collapse" state, resulting in $r_{mix} = \sqrt{9 - \sqrt{144q_0 - 63}}/2 \approx 0.757$, and $\varsigma = 5 \times 10^{-3}$.

The fact that we are using a Kadomtsev-like post-crash state does not limit our conclusions. We could have chosen the Gimblett-Hastie state [12], which differs from the one used in this work by having a Taylor-relaxed inner region, allowing for $q < 1$ around the axis.

Since Eq. (21) is of the heat diffusion type, in order to solve it we need an initial value over the whole domain $r \in [0, 1]$, and two boundary conditions at $r = 0$ and $r = 1$. The initial value is given by the function q_{pc} defined in Eq. (27). We then choose the Cauchy boundary condition $q(t, 1) = q_{in}(1)$, i.e. the total plasma current is held constant, and the Neumann boundary condition $\partial_r q^{-1}(t, 0) = 0$. The last one is chosen because we want our system to evolve, at very long times, towards an equilibrium magnetic field which is regular for $r \rightarrow 0$. To clarify this point, let us consider the case of constant resistivity. After setting $\partial_t \equiv 0$ and integrating Eq. (21) twice, one obtains the equilibrium safety factor

$$q_{eq}^{(0)}(r) = \frac{r^2}{C_2 + C_1 r^2}, \quad (28)$$

where $C_{1,2}$ are two constants of integration. Then, if $C_2 \neq 0$ for $r \rightarrow 0$, we have $q_{eq}(r) \rightarrow C_2^{-1}r^2$, which implies a divergent magnetic field, $B_\theta/B_0 \equiv B \sim C_2/r$ for $r \rightarrow 0$. Hence, we set $C_2 \equiv 0$. Then $q_{eq}(r) \rightarrow C_1^{-1}$ for $r \rightarrow 0$, which yields $B/r \sim C_1$ for $r \rightarrow 0$. From this it follows that $\partial_r(B/r) = 0$, or equivalently $\partial_r q^{-1}(t, 0) = 0$. The result in Eq. (28) can be easily generalised to the case of non-constant resistivity, giving

$$q_{eq}(r) = q_{pc}(r = 1) \left(\int_0^1 \frac{\varrho d\varrho}{\hat{\eta}(\varrho)} \right) \frac{1}{\int_0^r \frac{\varrho d\varrho}{\hat{\eta}(\varrho)}}. \quad (29)$$

The integrals in Eq. (29) can be performed analytically, then the result can be compared to the long time evolution of Eq. (21). Figure 7 shows this solution for $t/\tau_* = 120, 200, 300$ and the analytical solution $q_{eq}(r)$ for $\nu_* = 10^{-4}$; the analytical steady-state equilibrium is recovered. It is worth noticing that it is reached on a time which is much shorter than the resistive diffusion time, $\tau_\eta = 5 \times 10^4 \tau_*$. Another important property of the steady-state solution of Eq. (21) is that it must give a (radially) constant electric field [see Eq. (19)]. In Fig. 4b we show the electric field $E = \hat{\eta} r^{-1} \partial_r(r^2/q)$ calculated from the solutions in Fig. 4a. As expected, the final state is uniform through the domain of integration. Finally, in Fig. 5 we show an example of the evolution of the whole q profile, starting from the Kadomtsev post-crash state. The formation of the structure on the width δ_* is evident. It is also clear that a very fast diffusion of the initial current sheet occurs at the mixing radius r_{mix} . In tokamaks, such a fast neoclassical evolution of the q profile seems to occur only when approaching stationary conditions [13, 14]. Here we also note that the final equilibrium of Eq. (29) is ideal MHD unstable to the $m = 1, n = 2$ mode, (since $q_0 < 1/2$ [8]) so, from an operational point of view, it should never be achieved! Incidentally, all the q profiles resulting from this analysis are very similar to those proposed within the incomplete reconnection model of Ref. [1].

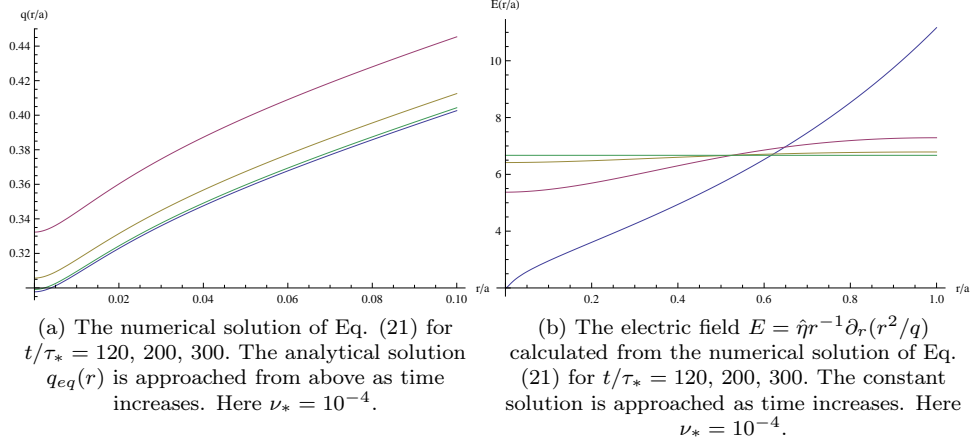


Figure 4: Long time evolution of Eq. (21) for q profile and electric field.

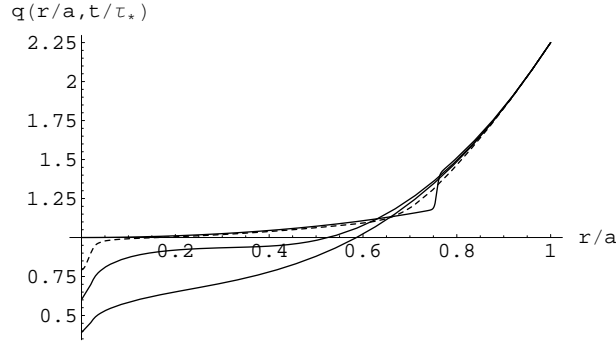


Figure 5: Solution of Eq. (21) for time $t/\tau_* = 0, 2.5, 37.5$, and 49.75 . Here $\nu_* = 0.001$.

B. Axial criterion

In the previous section we drew attention to the fact that q_0 undergoes a rapid downward evolution. One might consider what can limit such evolution of q on axis. It is known that, even before the ideal MHD $m = 1, n = 2$, instability becomes possible (when $q_0 < 1/2$), the tearing mode stability index $\Delta'_{m,n}$ for core resonant modes, such as $m = 2, n = 3$, or $m = 3, n = 4$, can become positive and potentially unstable, since diamagnetic stabilisation is likely to be extremely weak close to the magnetic axis, and average curvature is unfavourable [15]. Hence, it is tempting to look for a correlation between the onset of such modes, and the Sawtooth period. Since experimentally Sawteeth are observed to happen when $q_0 \approx 0.75$, [16] we solve Eq. (21) for several values of ν_* , ranging from $\nu_* = 10^{-4}$ to $\nu_* = .1$ and solve for the time at which $q(0) = 0.75$.

ν_*	τ_{SAW}/τ_*	ν_*	τ_{SAW}/τ_*
0.1	0.35	0.01	1.96
0.09	0.37	0.005	1.56
0.075	0.4	0.0025	2.43
0.05	0.49	0.001	4.75
0.03	0.62	0.0005	8.42
0.025	0.67	0.00025	15.6
0.015	0.09	0.0001	36.9

Table II: Values plotted in Fig. 6.

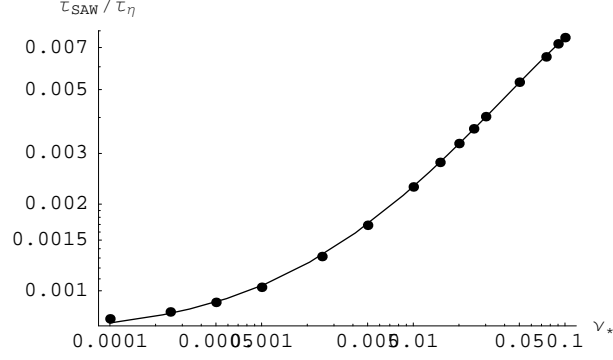


Figure 6: Data from Tab. II. A fit gives $\tau_{SAW}/\tau_\eta = 7 \times 10^{-4} + 0.034\nu_*^{2/3} + 0.004\nu_* - 0.08\nu_*^2$.

Figure 6 shows the results of Table II graphically. A fit of the data gives

$$\tau_{SAW}/\tau_\eta = 7 \times 10^{-4} + 0.034\nu_*^{2/3} + 0.004\nu_* - 0.08\nu_*^2. \quad (30)$$

From this it is evident that when $0.001 \lesssim \nu_* \lesssim 0.1$, $\tau_{SAW}/\tau_* \sim \nu_*^{-1/3}$, then

$$\tau_{SAW} \sim \tau_* \nu_*^{-1/3} \propto R_0^{8/3} N_e^{2/3} T_e^{1/6} \text{ sec}, \quad (31)$$

where we do not distinguish between the length R_0 and a , since JET and ITER share the same aspect ratio, $R_0/a = 3$. For smaller values of ν_* , τ_{SAW} scales as $\tau_* \nu_*^{-1}$, thus the ν_* dependence in τ_{SAW} disappears. This is equivalent to saying that the fast evolution of q on-axis is a transient phenomenon regularising the q profile. For very small values of ν_* such that $\delta_*/r_1 \ll 1$, the effect of ν_* on the global diffusion of the q profile is negligible. From a preliminary analysis, we also find that the presence of the Bootstrap current terms in Ohm's law reduces the strength of the electron trapping effect, i.e. the development of localised axial structures near axis.

These results have to be compared to other similar ones. For example, Park and Monticello [3], considering neoclassical diffusion within the $q = 1$ radius in nonlinear 3D MHD simulations, found $\tau_{SAW} \propto T_e^{3/2} R_0^2$. Another interesting result is that of McGuire and Robinson [17] who found $\tau_{SAW} \propto N_e^{3/7} T_e^{19/14}$ without, or $\tau_{SAW} \propto N_e^{3/5} T_e^{23/10}$ with diamagnetic effects; these were obtained with an empirical fit to data. Notice that the weak temperature dependence we found in Eq. (31) is mainly given by the different dependence of η and ν_* on temperature. In our

JET	ITER
$a = 1 \text{ m}$	$a = 3 \text{ m}$
$T_e = 4 \text{ keV}$	$T_e = 25 \text{ keV}$
$\tau_\eta \sim 400 \text{ sec}$	$\tau_\eta \sim 24 \times 10^3 \text{ sec}$
$\tau_* \sim 0.86 \text{ sec}$	$\tau_* \sim 3 \text{ sec}$
$\nu_* \sim 0.01$	$\nu_* \sim 6 \times 10^{-4}$
$\delta_* \sim 4.6 \text{ cm}$	$\delta_* \sim 1.4 \text{ cm}$

Table III: Resistive time τ_η , fast diffusive time τ_* , normalised electron collision frequency ν_* , and boundary layer δ_* for both JET and ITER.

case, when we consider the ratio τ_η/τ_* , we see that $\tau_\eta^{JET}/\tau_*^{JET} \sim 10^2$, and $\tau_\eta^{ITER}/\tau_*^{ITER} \sim 10^4$. If we take $R_0/a = 3$, and $N_e \sim 10^{20} \text{ m}^{-3}$ and compare the two machines, we obtain the results in Table III. By using the results in Table II and III, one obtains $\tau_{SAW}^{JET} \approx 1.69 \text{ sec}$, and $\tau_{SAW}^{ITER} \approx 25 \text{ sec}$. A sawtooth period of 1.7 sec. is the longest observed in JET, while 40 sec. is the value empirically allowed to avoid triggering Neoclassical Tearing Modes [18].

C. Time evolution of shear at the $q = 1$ rational surface

In Section 3, we stressed the fact that both $\hat{\beta}$ and Δ' , which are fundamental to the calculation of the boundary of marginal stability of the drift-tearing and kink modes, show a shear dependence. Hence, here we present the time

evolution of: r_1 , the position of the $q(r_1, t) = 1$ surface, the parameter $\hat{\beta} = 0.5/[aq'(r_1)]^2$, the shear at the $q = 1$ surface $\hat{s}_1 = r_1 q'(r_1)$, and finally, because of its relevance to core MHD stability, the value of q on axis.

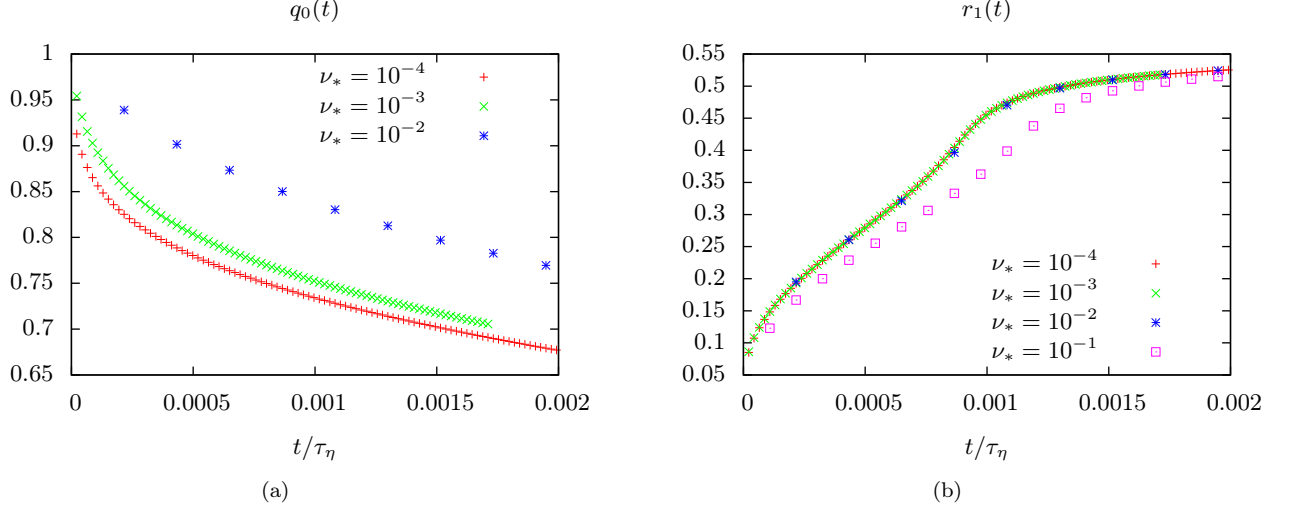


Figure 7: The value of q on axis and the position of the $q = 1$ surface calculated from Eq. (21) for different ν_* .

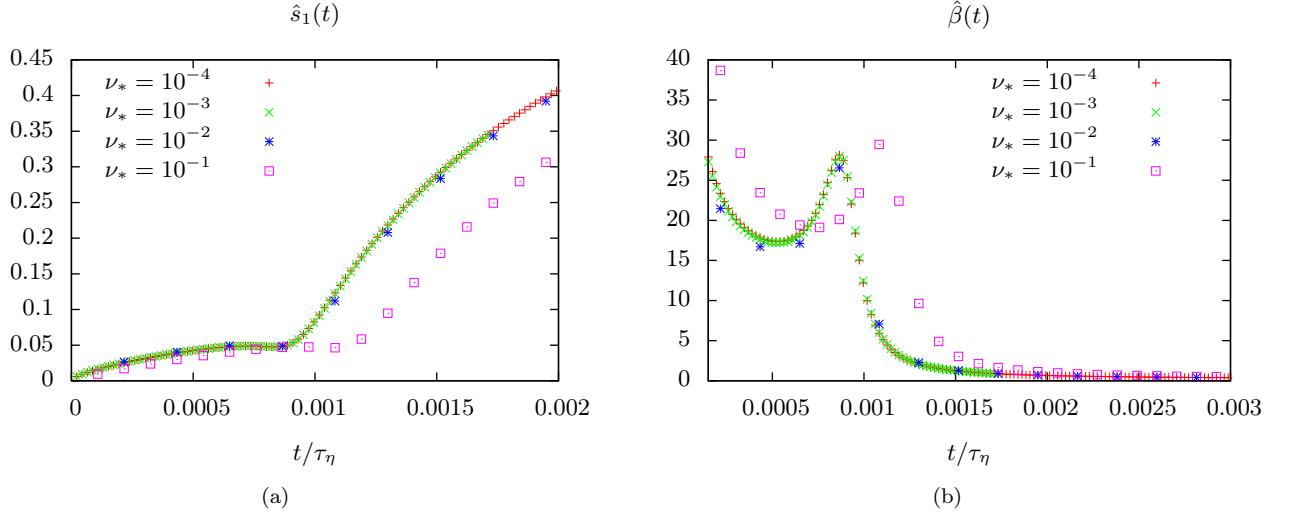


Figure 8: The shear at the resonant surface \hat{s}_1 and the parameter $\hat{\beta}$ calculated from Eq. (21) for different ν_* .

From transport modelling simulations, we know that a typical value of critical shear at which the sawtooth is expected to be triggered is $\hat{s}_1 = 0.4$ [19]. In our case, this is reached by the time the position of the rational surface r_1 has approached the value at which $q_{in} = 1$ in the initial Kadomstev pre-crash state.

IV. DISCUSSION AND CONCLUSIONS

In this work we have discussed the role of the dissipative, $m = 1, n = 1$, modes, in the prediction of the Sawtooth period in tokamaks, and explored the effect of neoclassical resistivity in determining the evolution of the plasma during the quiescent ramp phase of the Sawtooth.

In Ref. [2], we calculated the critical value of the stability index $\Delta'_{1,1}$ for crossing a linear stability threshold of the drift-tearing and dissipative kink modes, with gyrokinetic ions and semicollisional electrons.

The stability thresholds derived in Ref. [2] depend sensitively on the magnitude of certain plasma parameters at the $q = 1$ surface, such as the shear $\hat{s}_1(t)$, and $\hat{\beta}(r, t) = 0.5\beta_e L_s^2/L_n^2$. We have therefore explored the resistive evolution of these parameters. In addition, in order to address the possibility that the $m = 1, n = 1$ mode might actually be triggered by a core plasma instability near the magnetic axis, we have also monitored the evolution of $q_0(t)$.

For the parameters defining the linear stability threshold of the $m = 1, n = 1$ mode, [i.e. $r_1(t)$, $\hat{s}_1(t)$, and $\hat{\beta}(t)$,] we found negligible dependence on the collisionality parameter ν_* , but faster evolution [in agreement with [3]] than would occur with Spitzer resistivity. On this basis, one would expect a scaling of the Sawtooth period from JET to ITER proportional to $T_e^{3/2} a^2$, assuming that energetic ion stabilising contributions are similar in both cases.

However, if the rapid downward evolution of $q_0(t)$ were to be responsible for triggering a Sawtooth collapse, we find some sensitivity to the magnitude of ν_* and a weaker scaling of the Sawtooth period with T_e . These two different scalings may offer a means to distinguish the trigger mechanism in several machines. A suggested scaling from JET to ITER, in this scenario, is $\tau_{SAW} \propto R_0^{8/3} N_e^{2/3} T_e^{1/6}$.

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 - [20] We calculated the critical shear for instability from the condition (14) using the growth rate in Eq. (94) of Ref. [2]. In this way, we recover a scaling with Lundquist number, electron β_e and ion Larmor radius which is the same of Eq. (15a) of Ref. [1].
 - [21] The tanh functions have only been inserted to provide a slight spread of the initial current sheet of width ς at r_{mix} in the Kadomtsev model. The choice of the pre-crash is $q(r) = q_0/[1 - r^2 + (1/3)r^4]$. This is actually the steady-state q if resistivity were Spitzer and $T_e \propto (1 - r^2/a^2)^{4/3}$.